

Structured Model Reference Adaptive Control for a Class of Nonlinear Systems

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A control design methodology for a particular class of nonlinear dynamic systems in the structured first-order form is presented. The differential equations are distinctly categorized as two sets, one representing exact kinematic differential equations and the other representing an uncertain dynamic model. The control law seeks exponential stability of the position errors for the known parameter case. For the uncertain parameter case, an adaptive controller is designed that guarantees bounded position tracking errors. The efficacy of the controller is supported through simulation examples.

Introduction

OVER the past two decades, adaptive control formulations have evolved and have been studied as candidates for controlling low to moderate dimensioned uncertain dynamic systems. It is well known that the mathematical structure of the dynamic systems is affected by the choice of coordinates.^{1,2}

The basic theory of model reference adaptive control is quite mature, following significant contributions during the 1980s from Narendra and Annaswamy,³ Sastry and Bodson,⁴ and Ioannou and Sun.⁵ Adaptive control laws for various classes of nonlinear systems have been studied based on the concept of feedback linearization, wherein a parameter-independent diffeomorphism is first assumed to exist. Then, the transformed system, whether linear time invariant or otherwise, becomes amenable to application of the well-known adaptive control techniques⁶ in the new set of coordinates.

These strategies have been successfully studied with potential applications to aerospace vehicle control, especially in the area of attitude stabilization and tracking of spacecraft.^{7–12} Note that the main reason for the success in tackling the spacecraft and robot manipulator problems is the recognition of the clean structure implicit in the equations of motion that describe these systems. This structure is exploited by Slotine and DiBenedetto¹³ to derive a Hamiltonian adaptive controller. Ahmed, Coppola, and Bernstein¹⁰ present a novel parameterization of the inertia matrix entries and utilize the structure to achieve asymptotic tracking of spacecraft attitude maneuvers, in addition to identifying the inertia parameters. Wong, Queiroz, and Kapila¹¹ use the same inertia matrix parameterization and combine a filter that estimates the angular rates of a spacecraft using attitude measurements to track spacecraft maneuvers. Zegeroglu et al.¹⁴ discuss the solution of the nonlinear tracking of kinematically redundant robot manipulators, and one can find an excellent summary of the similarities between the robot manipulator problem and the spacecraft control problem in Ref. 15. Furthermore, the nature of the equations of motion governing dynamic systems in the second-order form also renders the system amenable to passivity-based control strategies.^{9,12} Several output feedback-based stabilization techniques with and without adaptation have been proposed.

However, the preceding does not translate easily to the aircraft problem, principally because of the implicit dependence of the aerodynamic parameters on the states of the system. In a spacecraft attitude maneuver problem, the net change in inertia due to the propellant used in the case of thrusters is very low compared to the total mass of the system. Furthermore, one could always use momentum wheels or passive devices that do not change the mass properties drastically. Thus, the inertia parameters remain essentially constant over the maneuver period. The system matrices of importance in the aircraft problem are those that arise out of the parameterization of the aerodynamic and propulsive forces and moments. These matrices do not have any prescribed structure and, as a result, one can not use the “nice” properties that have been exploited so well in the spacecraft problem.

Singh and Steinberg⁶ discuss the application of adaptive feedback linearization applied to flight control, where the reference maneuvers are generated through a command generator in the absence of external disturbances. The present paper extends the results to an application for tracking an aggressive 9-g, 180-deg heading change maneuver at constant altitude and velocity in the presence of external disturbances and initial condition errors.

In this paper, we start with a nonlinear parameterization^{16–18} of the system dynamics and derive control laws that guarantee stable tracking of reference trajectories. In the first section, the governing differential equations describing the system dynamics, and the assumptions therein, are explained in detail. The second section deals with the derivation of the control law for the known parameter case, whereas the third section outlines the derivation of the control law for the uncertain parameter case. A class of bounded disturbances and their effects on the closed-loop properties of the overall system is discussed in the fourth section, and the fifth section highlights the applicability of this approach through a challenging example, followed by a brief summary of the results from the development.

System Description

We consider nonlinear dynamic systems, affine in control, that can be categorized as two sets of differential equations, one representing the exact kinematics and the other representing the uncertain dynamics. These sets of differential equations can be compactly written, as in the following structured state–space form:

$$\dot{\sigma} = J(\sigma)\omega \quad (1)$$

$$\dot{\omega} = Ag(\sigma, \omega) + Bu + H(\sigma, \omega) + d \quad (2)$$

where $\sigma \in \mathcal{R}^n$ is the vector of position coordinates; $\omega \in \mathcal{R}^n$ is the vector of velocity coordinates; $J(\sigma) \in \mathcal{R}^{n \times n}$ is the nonlinear transformation matrix relating $\dot{\sigma}$ and ω ; $A \in \mathcal{R}^{n \times p}$ is a matrix of system parameters; $g(\sigma, \omega) \in \mathcal{R}^p$ is a vector of smooth, regular nonlinear functions used to describe the unforced dynamic behavior; $B \in \mathcal{R}^{n \times n}$ is the control influence matrix; $u \in \mathcal{R}^n$ is the vector of

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control inputs (under the assumption that the number of control inputs is equal to the number of momentum states); $H(\sigma, \omega) \in \mathcal{R}^n$ is a vector of known nonlinear functions that do not depend on any system parameters or, even if they do, they are known to a sufficiently high degree of accuracy (for example, kinematic coupling terms); and $d \in \mathcal{R}^n$ is a vector of bounded disturbances.

Control Law Derivation for the Known Parameter Case

Let the reference model to be tracked be described by a similar set of known equations analogous to the system description in Eqs. (1) and (2). The reference model is, therefore, represented as follows:

$$\dot{\sigma}_r = J(\sigma_r)\omega_r \quad (3)$$

$$\dot{\omega}_r = A_r g(\sigma_r, \omega_r) + B_r u_r + H(\sigma_r, \omega_r) \quad (4)$$

The reference model defined in Eqs. (3) and (4) is assumed to be the best currently available information about the actual system, which, for example, could be the output of an on-the-fly system identification process or an a priori model. Thus, given the control requirements, Eqs. (3) and (4) have been solved for a reference trajectory $\sigma_r(t)$, $\omega_r(t)$, and $u_r(t)$.

Let $s \triangleq \sigma - \sigma_r$ be the position tracking error. The objective of the control problem is to seek to drive $\dot{s} + \lambda s \rightarrow 0$ for $\lambda > 0$ as $t \rightarrow \infty$, which implies, in turn, that both $s \rightarrow 0$ and $\dot{s} \rightarrow 0$ as $t \rightarrow \infty$ (because $\dot{s} + \lambda s \in \mathcal{L}_2$, that is, $\dot{s} + \lambda s$ has a finite two-norm) for the case when full state information is available for feedback.

Let $x \triangleq \omega - \omega_r$ be the velocity tracking error. Then the tracking error dynamics can be written as follows:

$$\dot{s} = J(\sigma)\omega - J(\sigma_r)\omega_r = F \quad (5)$$

$$\begin{aligned} \dot{x} = & Ag(\sigma, \omega) + Bu + H(\sigma, \omega) + d \\ & - A_r g(\sigma_r, \omega_r) - B_r u_r - H(\sigma_r, \omega_r) \end{aligned} \quad (6)$$

Let us add and subtract ϕ , an undetermined function to be specified later, and add and subtract a stabilizing term $(A_m x - A_m x)$ to the right-hand side of Eq. (6). With these modifications, Eq. (6) can be rewritten as

$$\begin{aligned} \dot{x} = & A_m x + \phi + Ag(\sigma, \omega) + Bu + H(\sigma, \omega) + d \\ & - A_r g(\sigma_r, \omega_r) - B_r u_r - H(\sigma_r, \omega_r) - A_m x - \phi \end{aligned} \quad (7)$$

We choose A_m to be any Hurwitz matrix, that is, all eigenvalues of A_m lie in the open left half-plane. For the case when $d = 0$, defining $\psi = -H(\sigma, \omega) + A_r g(\sigma_r, \omega_r) + B_r u_r + H(\sigma_r, \omega_r) + A_m x + \phi$, we can compactly write Eq. (7) as

$$\dot{x} = A_m x + \phi + [Bu + Ag(\sigma, \omega) - \psi] \quad (8)$$

Observe that because the number of control inputs are equal to the number of momentum states, B will be of full rank, and the inverse can be computed. We can now choose the control law to be

$$u = -B^{-1}[Ag(\sigma, \omega) - \psi] \quad (9)$$

By substituting the preceding control law in Eq. (8), we obtain the overall closed-loop tracking error dynamics of the form

$$\dot{x} = A_m x + \phi \quad (10)$$

$$\dot{s} = F \quad (11)$$

In the preceding system, we then treat the as-yet-unspecified function ϕ as the control input to achieve the necessary tracking control objective.

When a modified tracking error variable, $y \triangleq \dot{s} + \lambda s = J(\sigma)[x + \tilde{s}]$, is defined, where $\tilde{s} = (I - J(\sigma)^{-1}J(\sigma_r))\omega_r + J(\sigma)^{-1}(\lambda s)$, it is noted that $\dot{J}(\sigma)^{-1} = -J(\sigma)^{-1}\dot{J}(\sigma)J(\sigma)^{-1}$ and $\dot{\tilde{s}} = \dot{\omega}_r - J(\sigma)^{-1}\{J(\sigma_r)\dot{\omega}_r + \dot{J}(\sigma_r)\omega_r - J(\sigma)J(\sigma)^{-1}[J(\sigma_r)\omega_r - \lambda s] - \lambda \dot{s}\}$.

The objective now is to define the function ϕ , such that the modified tracking error dynamics takes the form $\dot{y} = A_m y$. To achieve that, let us define a candidate Lyapunov function $V(y)$ as

$$V = y^T P y, \quad P = P^T > 0 \quad (12)$$

Clearly, $V(y) = 0$ only if $y = 0$, or, equivalently, $\dot{s} + \lambda s = 0$. Exponential convergence of the tracking error y to zero can be achieved

by choosing the function ϕ as

$$\phi = J(\sigma)^{-1} \left\{ A_m y - \frac{d}{dt}[J(\sigma)](x + \tilde{s}) \right\} - A_m x - \dot{\tilde{s}} \quad (13)$$

The preceding can be derived by the differentiation of y (defined earlier), the choice of ϕ to nullify all of the nonlinear terms, and the enforcement of $\dot{y} = A_m y$.

Note that $\dot{\tilde{s}}$ can be computed analytically, and the computation involves only σ , σ_r , ω , and ω_r . The design of ϕ renders the modified tracking error dynamics as an exponentially stable system, which guarantees that $y \rightarrow 0$ as $t \rightarrow \infty$ or $\dot{s} + \lambda s \rightarrow 0$. That is, $s \rightarrow 0$ and $\dot{s} \rightarrow 0$ as $t \rightarrow \infty$, which, in turn, implies that $\sigma \rightarrow \sigma_r$ and $\omega \rightarrow \omega_r$. Thus, the control objective is achieved.

Control Law Derivation for the Uncertain Parameter Case

For the unknown parameter case, the control law proposed in the earlier section [Eq. (9)] can not be used because A and B are poorly known. We continue with the formulation as earlier, following the definition of ϕ , such that the error dynamics is represented by

$$\dot{s} = F \quad (14)$$

$$\dot{x} = A_m x + \phi + [Bu + Ag(\sigma, \omega) - \psi] \quad (15)$$

Defining ϕ as in Eq. (13), we can combine Eqs. (14) and (15) to obtain

$$\dot{y} = A_m y + J(\sigma)[Bu + Ag(\sigma, \omega) - \psi] \quad (16)$$

Obviously, when the parameters are all known, the choice of $u = -B^{-1}[Ag(\sigma, \omega) - \psi]$ leads to the same conclusions as in the earlier section. However, when the parameters are uncertain, the control law is implemented as

$$u = -\hat{B}^{-1}[\hat{A}g(\sigma, \omega) - \psi] \quad (17)$$

where, \hat{A} and \hat{B} are the current estimates of A and B , respectively, obtained from some adaptive law that has yet to be determined. Rearrangement of the preceding yields the control identity $\hat{B}u + \hat{A}g(\sigma, \omega) - \psi = 0$. Using this identity, that is, subtracting it within [] in Eq. (16), we obtain

$$\dot{y} = A_m y - J(\sigma)[\tilde{B}u + \tilde{A}g(\sigma, \omega)] \quad (18)$$

where, $\tilde{B} = \hat{B} - B$ and $\tilde{A} = \hat{A} - A$. The following analysis leads to the development of the adaptive laws. Let us define a candidate Lyapunov function $V(y, \tilde{A}, \tilde{B})$ as follows:

$$V(y, \tilde{A}, \tilde{B}) = y^T P y + T r [\tilde{A}^T \Gamma_1^{-1} \tilde{A} + \tilde{B}^T \Gamma_2^{-1} \tilde{B}] \quad (19)$$

where, $P = P^T > 0$ and $\Gamma_i = \Gamma_i^T > 0$, $i = 1, 2$. Clearly, $V(y, \tilde{A}, \tilde{B}) = 0$, only when $y = 0$, $\tilde{A} = 0$, and $\tilde{B} = 0$. Differentiating Eq. (19) with respect to time and evaluating \dot{V} along the trajectories of Eq. (18), we can show that

$$\dot{V} = -y^T R y \quad (20)$$

by choosing the adaptive laws for \hat{A} and \hat{B} as

$$\dot{\hat{A}} = \Gamma_1 J(\sigma)^T P y g(\sigma, \omega)^T \quad (21)$$

$$\dot{\hat{B}} = \Gamma_2 J(\sigma)^T P y u^T \quad (22)$$

$R = R^T > 0$, such that the existence of P is always guaranteed, because A_m is Hurwitz. P is obtained by solving the Lyapunov equation $PA_m + A_m^T P = -R$. Note that the control law implementation requires the inverse of \hat{B} , and so instead of inverting \hat{B} at every instant, we could actually implement the control law using the adaptive estimate of the inverse of \hat{B} . This is done with the following identity: $(d/dt)[\hat{B}^{-1}] = -M(d/dt)[\hat{B}]M$ where, $M = \hat{B}^{-1}$.

Closed-Loop Stability Analysis

The preceding adaptive control law is globally stable under a reasonable set of circumstances that we develop hereafter, using the approach established in Refs. 5, 19, and 20. From the properties of V and \dot{V} , we see that $\mathbf{y} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{\mathbf{A}}, \dot{\mathbf{B}} \in \mathcal{L}_\infty$.

Lemma: If $\mathbf{y} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ then $\dot{\mathbf{s}} = -\lambda \mathbf{s} + \mathbf{y} \Rightarrow \mathbf{s}, \dot{\mathbf{s}} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ for $\lambda > 0$.

Proof: If \mathbf{s} is input-to-state stable with respect to input \mathbf{y} and if in addition, $\mathbf{y} \rightarrow 0$, then \mathbf{s} and $\dot{\mathbf{s}} \rightarrow 0$. For proof see Ref. 21.

Theorem: For the system described by Eqs. (1) and (2) ($d = 0$) and the reference trajectories generated by Eqs. (3) and (4), the control law in Eq. (17), together with the adaptive law in Eqs. (21) and (22), ensures that $\boldsymbol{\sigma} \rightarrow \boldsymbol{\sigma}_r$ and $\boldsymbol{\omega} \rightarrow \boldsymbol{\omega}_r$.

Proof: Because \mathbf{s} and $\dot{\mathbf{s}} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$, and $\boldsymbol{\sigma}_r$ and $\dot{\boldsymbol{\sigma}}_r \in \mathcal{L}_\infty$, we conclude that $\boldsymbol{\sigma}$ and $\dot{\boldsymbol{\sigma}} \in \mathcal{L}_\infty$. Therefore, using Eq. (1), we conclude that $\boldsymbol{\omega} \in \mathcal{L}_\infty$. This implies that $\mathbf{g}(\boldsymbol{\sigma}, \boldsymbol{\omega}) \in \mathcal{L}_\infty$ and $\mathbf{H}(\boldsymbol{\sigma}, \boldsymbol{\omega}) \in \mathcal{L}_\infty$, all of which leads us to conclude that $\boldsymbol{\psi} \in \mathcal{L}_\infty$.

Because $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}} \in \mathcal{L}_\infty$, we can conclude from Eq. (18) that $\dot{\mathbf{y}} \in \mathcal{L}_\infty$. Because $\mathbf{y} \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\dot{\mathbf{y}} \in \mathcal{L}_\infty$, using Barbalat's lemma (see Ref. 5), we conclude that $\mathbf{y} \rightarrow 0$ as $t \rightarrow \infty$.

Because $\mathbf{y} = \dot{\mathbf{s}} + \lambda \mathbf{s} \rightarrow 0$, as $t \rightarrow \infty$, it is trivial to show that $\mathbf{s} \rightarrow 0$ and $\dot{\mathbf{s}} \rightarrow 0$ as $t \rightarrow \infty$. Hence, $\boldsymbol{\sigma} \rightarrow \boldsymbol{\sigma}_r$ and $\dot{\boldsymbol{\sigma}} \rightarrow \dot{\boldsymbol{\sigma}}_r$ or $\boldsymbol{\omega} \rightarrow \boldsymbol{\omega}_r$ as $t \rightarrow \infty$. This concludes the proof. ■

Structured Adaptive Control in the Presence of Bounded Disturbances

In the presence of a bounded disturbance vector, the preceding results can be extended, and the tracking error dynamics can be written as

$$\dot{\mathbf{y}} = \mathbf{A}_m \mathbf{y} - \mathbf{J}(\boldsymbol{\sigma})[\tilde{\mathbf{B}}\mathbf{u} + \tilde{\mathbf{A}}\mathbf{g}(\boldsymbol{\sigma}, \boldsymbol{\omega})] + \mathbf{J}(\boldsymbol{\sigma})\mathbf{d} \quad (23)$$

It is well known that if the effect of the disturbance is not accounted for during the adaptive law design, the adaptive law can become unstable due to parameter drift. We can find several techniques in the literature⁵ to ensure that this instability phenomenon is avoided, such as the fixed- ν modification, switching- ν modification, etc. However, the choice of each technique has certain drawbacks associated with it. We conclude that the choice of the technique depends on the problem under consideration. In the present problem, we shall consider the fixed- ν modification to avoid this instability phenomenon. To this effect, we modify the adaptive law derived earlier as follows:

$$\dot{\hat{\mathbf{A}}} = \Gamma_1 \mathbf{J}(\boldsymbol{\sigma})^T \mathbf{P} \mathbf{y} \mathbf{g}(\boldsymbol{\sigma}, \boldsymbol{\omega})^T - (\nu/2) \hat{\mathbf{A}} \quad (24)$$

$$\dot{\hat{\mathbf{B}}} = \Gamma_2 \mathbf{J}(\boldsymbol{\sigma})^T \mathbf{P} \mathbf{y} \mathbf{u}^T - (\nu/2) \hat{\mathbf{B}} \quad (25)$$

where ν will be defined later. The addition of the second term in the adaptive law is to ensure that the time derivative of the Lyapunov function used to analyze the adaptive scheme becomes negative in the space of the parameter estimates, when these parameters exceed certain bounds. Thus, the derivative of the Lyapunov function is now modified as follows:

$$\begin{aligned} \dot{V} = & -\mathbf{y}^T \mathbf{R} \mathbf{y} + \mathbf{y}^T \mathbf{P} \mathbf{J}(\boldsymbol{\sigma})\mathbf{d} + \mathbf{d}^T \mathbf{J}(\boldsymbol{\sigma})^T \mathbf{P} \mathbf{y} \\ & - \nu \text{Tr}(\tilde{\mathbf{A}}^T \Gamma_1^{-1} \hat{\mathbf{A}}) - \nu \text{Tr}(\tilde{\mathbf{B}}^T \Gamma_2^{-1} \hat{\mathbf{B}}) \end{aligned} \quad (26)$$

Manipulating the terms on the right-hand side (RHS) as outlined in Ref. 22, we can show

$$\text{RHS} \leq -\frac{1}{2} \mathbf{y}^T \mathbf{R} \mathbf{y} + \frac{1}{2} \mathbf{d}^T \mathbf{J}(\boldsymbol{\sigma})^T \mathbf{R}_d \mathbf{J}(\boldsymbol{\sigma}) \mathbf{d}$$

where $\mathbf{P} \mathbf{A}_m^{-1} + \mathbf{A}_m^{-T} \mathbf{P} = -\mathbf{R}_d$.

Furthermore, one can show that

$$\begin{aligned} \dot{V} \leq & -\alpha V - \frac{1}{2} \mathbf{y}^T (\mathbf{R} - 2\alpha \mathbf{P}) \mathbf{y} - [(v/2) - \alpha] [\text{Tr}(\tilde{\mathbf{A}}^T \Gamma_1^{-1} \tilde{\mathbf{A}}) \\ & + \text{Tr}(\tilde{\mathbf{B}}^T \Gamma_2^{-1} \tilde{\mathbf{B}})] + (v/2) [\text{Tr}(\mathbf{A}^T \Gamma_1^{-1} \mathbf{A}) \\ & + \text{Tr}(\mathbf{B}^T \Gamma_2^{-1} \mathbf{B})] + \frac{1}{2} \mathbf{d}^T \mathbf{J}(\boldsymbol{\sigma})^T \mathbf{R}_d \mathbf{J}(\boldsymbol{\sigma}) \mathbf{d} \end{aligned}$$

where $\alpha > 0$. If we choose $\alpha = \min[\frac{1}{2} \mu(\mathbf{R}) \mu(\mathbf{P}^{-1}), \nu/2]$, where $\mu(\cdot)$ is the spectral radius (maximum eigenvalue) operator, then it immediately follows that

$$\begin{aligned} \dot{V} \leq & -\alpha V + \frac{1}{2} \mathbf{d}^T \mathbf{J}(\boldsymbol{\sigma})^T \mathbf{R}_d \mathbf{J}(\boldsymbol{\sigma}) \mathbf{d} \\ & + (v/2) [\text{Tr}(\mathbf{A}^T \Gamma_1^{-1} \mathbf{A}) + \text{Tr}(\mathbf{B}^T \Gamma_2^{-1} \mathbf{B})] \end{aligned} \quad (27)$$

If $\mathbf{d}_0 > 0$, an upper bound for the error term $\mathbf{J}(\boldsymbol{\sigma})\mathbf{d}$ for all $\boldsymbol{\sigma}(t)$ can be found, then $\dot{V} \leq 0$ whenever

$$\begin{aligned} V \geq V_0 \triangleq & (1/\alpha) \left\{ \frac{1}{2} \mathbf{d}_0^T \mathbf{R}_d \mathbf{d}_0 + (v/2) [\text{Tr}(\mathbf{A}^T \Gamma_1^{-1} \mathbf{A}) \right. \\ & \left. + \text{Tr}(\mathbf{B}^T \Gamma_2^{-1} \mathbf{B})] \right\} \end{aligned} \quad (28)$$

Note that the term $\frac{1}{2} \mathbf{d}^T \mathbf{J}(\boldsymbol{\sigma})^T \mathbf{R}_d \mathbf{J}(\boldsymbol{\sigma}) \mathbf{d}$ depends on $\boldsymbol{\sigma}$ and could be unbounded, especially when $\boldsymbol{\sigma}$ approaches any singular attitude configuration. Thus, it is required that $\mathbf{J}(\boldsymbol{\sigma})\mathbf{d}$ be bounded for achievement of the control objectives in the presence of disturbances. This essential requirement is missed completely if one starts with a linearized description of the kinematics. Also note that this quantity depends on the choice of the position coordinates. As an example, we look at the parameterization of the attitude using the modified Rodrigues parameters (see Ref. 17). For this case, $\mathbf{J}(\boldsymbol{\sigma})^T \mathbf{J}(\boldsymbol{\sigma}) = (1 + \boldsymbol{\sigma}^2)^2 \mathbf{I}$, where $\boldsymbol{\sigma}^2 \equiv \boldsymbol{\sigma}^T \boldsymbol{\sigma}$. If one was to represent the kinematics using the quaternions, then $\mathbf{J}(\boldsymbol{\sigma})\mathbf{d}$ is bounded; however, the effect of the additional constraint imposed by the quaternions needs to be investigated further.

Thus, we conclude that for cases when the kinematic differential equations approach singular configurations, the control law cannot accommodate disturbances, even if the disturbances are bounded. However, if the kinematics is guaranteed to be nonsingular, then an upper bound can always be found to guarantee stable tracking error dynamics.

Furthermore, because the disturbance is essentially unknown, the control law does not try to cancel the disturbance in any way.

Therefore, with the ν modification, we managed to extend the properties of the adaptive law for the ideal case when $\mathbf{d} = 0$, that is, $\mathbf{y}, \hat{\mathbf{A}}, \tilde{\mathbf{A}},$ and $\hat{\mathbf{B}} \in \mathcal{L}_\infty$ to the nonideal case when $\mathbf{d} \neq 0$ provided that $\mathbf{J}(\boldsymbol{\sigma})\mathbf{d} \in \mathcal{L}_\infty$. If the preceding can be guaranteed, then, by integrating Eq. (28), we can establish that the parameter errors converge exponentially to the residual set defined by $D_\nu = ((\tilde{\mathbf{A}}, \tilde{\mathbf{B}}) \in \mathcal{R}^{n \times p} \times \mathcal{R}^{n \times n} | \text{Tr}(\tilde{\mathbf{A}}^T \Gamma_1^{-1} \tilde{\mathbf{A}} + \tilde{\mathbf{B}}^T \Gamma_2^{-1} \tilde{\mathbf{B}}) \leq (1/\alpha) \{ \frac{1}{2} \mathbf{d}_0^T \mathbf{R}_d \mathbf{d}_0 + (v/2) [\text{Tr}(\mathbf{A}^T \Gamma_1^{-1} \mathbf{A}) + \text{Tr}(\mathbf{B}^T \Gamma_2^{-1} \mathbf{B})] \})$. However, the \mathcal{L}_2 property of $\mathbf{y}, \hat{\mathbf{A}},$ and $\hat{\mathbf{B}}$ can no longer be guaranteed in the presence of the nonzero error term $\mathbf{J}(\boldsymbol{\sigma})\mathbf{d}$, though it is possible to show that $\mathbf{y}, \hat{\mathbf{A}},$ and $\hat{\mathbf{B}}$ are $(\mathbf{d}_0^2 + \nu)$ -small in the mean-square sense. This is the price we pay by employing the fixed- ν modification to achieve robustness. If the bounds on the parameters are known a priori, one could use the switching- ν modification or parameter projection to achieve robustness and retain some of the stability properties. Furthermore, note that the mean square small bound on \mathbf{y} can lead to the phenomenon called bursting, that is, \mathbf{y} may assume values higher than the order of the modeling error/disturbance for short finite intervals of time. One way to avoid this bursting phenomenon is to use a dead-zone modification.

For the case when the disturbances are absent, one can choose $\nu = 0$ in the adaptive laws just derived. For this case, using the analysis in Ref. 5, we can show that $\mathbf{y} \rightarrow 0$, $\boldsymbol{\sigma} \rightarrow \boldsymbol{\sigma}_r$, and $\boldsymbol{\omega} \rightarrow \boldsymbol{\omega}_r$ as $t \rightarrow \infty$.

Consider the adaptive error equations and the error dynamics represented by Eqs. (21), (22), and (18). The tracking error dynamics is

$$\dot{\mathbf{y}} = \mathbf{A}_m \mathbf{y} - \mathbf{J}(\boldsymbol{\sigma})[\tilde{\mathbf{B}}\mathbf{u} + \tilde{\mathbf{A}}\mathbf{g}(\boldsymbol{\sigma}, \boldsymbol{\omega})] + \mathbf{J}(\boldsymbol{\sigma})\mathbf{d} \quad (29)$$

When $\tilde{\Theta} = [\tilde{\mathbf{A}}; \tilde{\mathbf{B}}]$ and $\tilde{\Psi} = [\mathbf{g}(\boldsymbol{\sigma}, \boldsymbol{\omega})^T \mathbf{u}^T]^T$ are denoted, Eq. (29) can be rewritten as

$$\dot{\mathbf{y}} = \mathbf{A}_m \mathbf{y} + \mathbf{J}(\boldsymbol{\sigma})(-\tilde{\Theta} \tilde{\Psi} + \mathbf{d}) \quad (30)$$

or $[\mathbf{y}] = \mathbf{W}_m(p)[\mathbf{J}(\boldsymbol{\sigma})\{-\tilde{\Theta} \tilde{\Psi} + \mathbf{d}\}]$, where $\mathbf{W}_m(p)$ is a diagonal matrix of rational stable transfer functions generated from \mathbf{A}_m . When

$\mathbf{d}_0 = J(\boldsymbol{\sigma})\mathbf{d}$ is defined, the adaptive laws can now be written as $\dot{\hat{\Theta}} = \Gamma J(\boldsymbol{\sigma})^T P \mathbf{y} \bar{\Psi}^T$. Because $\sup_t |\mathbf{d}_0(t)| \leq \mathbf{d}_m$, it follows that if $|\mathbf{y}| \gg \mathbf{d}_m$, then the signal $J(\boldsymbol{\sigma})(-\hat{\Theta}\bar{\Psi})$ is dominant in \mathbf{y} . On the other hand, if $|\mathbf{y}| < \mathbf{d}_m$, then \mathbf{d}_0 may be dominant in \mathbf{y} . Therefore, a small \mathbf{y} relative to \mathbf{d}_m indicates the possible presence of a large noise (because of \mathbf{d}_0) to signal $[J(\boldsymbol{\sigma})(-\hat{\Theta}\bar{\Psi})]$ ratio, whereas large \mathbf{y} relative to \mathbf{d}_m indicates a small noise to signal ratio. Thus, it seems reasonable to update the parameter estimate $\hat{\Theta}$ only when the signal $J(\boldsymbol{\sigma})(-\hat{\Theta}\bar{\Psi})$ in \mathbf{y} is large relative to the disturbance \mathbf{d}_0 . This method of adaptation is referred to as the dead zone because of the presence of a zone or interval where $\hat{\Theta}$ is constant, that is, no updating occurs. Thus, the adaptive law becomes

$$\begin{aligned} \dot{\hat{\Theta}} &= \Gamma J(\boldsymbol{\sigma})^T P (\mathbf{y} + \mathbf{y}_{dz}) \bar{\Psi}^T \\ \mathbf{y}_{dz} &= \begin{cases} 0 & \text{if } |\mathbf{y}| \geq \mathbf{y}_{dz}^0 \\ -\mathbf{y} & \text{if } |\mathbf{y}| < \mathbf{y}_{dz}^0 \end{cases} \end{aligned} \quad (31)$$

where, \mathbf{y}_{dz}^0 is a known strict upper bound for $\mathbf{d}_0(t)$, that is, $\mathbf{y}_{dz}^0 > \mathbf{d}_m \geq \sup_t |J(\boldsymbol{\sigma})\mathbf{d}(t)|$. The function $\pi(\mathbf{y}) = \mathbf{y} + \mathbf{y}_{dz}$ is known as the dead-zone function. It follows from Eq. (31) that for small \mathbf{y} , we have $\dot{\hat{\Theta}} = 0$, and no adaptation takes place, whereas for large \mathbf{y} , we adapt the same way as if there were no disturbance. Because the implementation of the dead zone as described could lead to problems related to the existence and uniqueness of solutions, as well as computational problems at the switching surface²³ because of the discontinuity of the dead-zone function, we avoid these problems using a continuous dead zone as defined here:

$$\begin{aligned} \dot{\hat{\Theta}} &= \Gamma J(\boldsymbol{\sigma})^T P (\mathbf{y} + \mathbf{y}_{dz}) \bar{\Psi}^T \\ \mathbf{y}_{dz} &= \begin{cases} -\mathbf{y}_{dz}^0 & \text{if } \mathbf{y} > \mathbf{y}_{dz}^0 \\ \mathbf{y}_{dz}^0 & \text{if } \mathbf{y} < -\mathbf{y}_{dz}^0 \\ -\mathbf{y} & \text{if } |\mathbf{y}| \leq \mathbf{y}_{dz}^0 \end{cases} \end{aligned} \quad (32)$$

Hence, the adaptive law with dead zone guarantees the property $\mathbf{y}, \hat{\Theta} \in \mathcal{L}_\infty$ in the presence of nonzero $\mathbf{d}_0 \in \mathcal{L}_\infty$. Also note that the dead zone preserves the \mathcal{L}_2 properties of the adaptive law, despite the presence of the bounded disturbance \mathbf{d}_0 . A lot of interesting properties of the dead zone can be inferred with the analysis outlined in Ref. 5. However, it is omitted here for the lack of space.

Simulation Example: High-g Turn Maneuver of an F-A/18-Like Aircraft

The simplified nonlinear equations of motion of a generic high-performance aircraft can be summarized as follows:

$$\begin{aligned} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} &= \begin{pmatrix} p + q \tan \theta \sin \phi + r \tan \theta \cos \phi \\ q \cos \phi - r \sin \phi \\ q \sin \phi \sec \theta + r \cos \phi \sec \theta \end{pmatrix} \\ \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \\ \dot{\alpha} \\ \dot{\beta} \end{pmatrix} &= \begin{pmatrix} l_\beta \beta + l_q q + l_r r + (l_{\beta\alpha} \beta + l_{r\alpha} r) \Delta \alpha + l_p p - i_1 q r \\ \bar{m}_\alpha \Delta \alpha + \bar{m}_q q + i_2 p r - m_\alpha p \beta + m_\alpha (g_0/V) (\cos \theta \cos \phi - \cos \theta_0) \\ n_\beta \beta + n_r r + n_p p + n_{p\alpha} p \Delta \alpha - i_3 p q + n_q q \\ q - p \beta + z_\alpha \Delta \alpha + (g_0/V) (\cos \theta \cos \phi - \cos \theta_0) \\ y_\beta \beta + p (\sin \alpha_0 + \Delta \alpha) - r \cos \alpha_0 + (g_0/V) \cos \theta \sin \phi \end{pmatrix} \end{aligned}$$

$$+ \begin{pmatrix} l_{\delta a} & l_{\delta r} & 0 \\ 0 & 0 & \bar{m}_{\delta e} \\ n_{\delta a} & n_{\delta r} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta_a \\ \delta_r \\ \delta_e \end{pmatrix}$$

where, α and β are the angle of attack and the angle of sideslip in the stability (wind) axes; ϕ, θ , and ψ are the roll, pitch, and yaw angles; p, q , and r are the body axes roll, pitch, and yaw rates; and δ_a, δ_r , and δ_e are the aileron, rudder, and elevator (stabilator) deflections, respectively. The terms $l_\alpha, m_\alpha, \bar{m}_\alpha, n_\alpha, z_\alpha$, and y_α are the stability axes derivatives or the aerodynamic influence terms due to the aircraft states and controls. Finally, i_α are the terms arising out of the aircraft's moments of inertia. This model represents the aircraft dynamics for a zero bank angle flight. The only nonlinearities that are retained in the dynamic model are the kinematic coupling and the inertial coupling terms.

We observe, in the preceding set of equations, that the control influence matrix is not full rank as assumed in the original problem formulation. However, looking at the $\dot{\alpha}$ equation and the $\dot{\beta}$ equation, we note that these variables can be controlled indirectly using the pitch rate and the yaw rate, respectively. Though y_β and z_α could be (in fact, will be) uncertain, they are usually well known within a sufficient degree of accuracy. Furthermore, the sign of these stability derivatives is accurately known. Because the α and β subsystems can be independently controlled using the body axis pitch rate and yaw rate, respectively, we could derive the desired rates that would be needed to stabilize these subsystems. These desired rates are then used to modify the reference pitch rate and yaw rate commands,^{6,24} thus ensuring reasonable dynamic behavior of α and β . If y_β and z_α are exactly known, one can implement the tracking control law to track $p_{\text{ref}}, q_{\text{ref}}$, and r_{ref} perfectly, and thereby achieve perfect tracking of α_{ref} and β_{ref} . Because the maneuver to be tracked in the case under study is a high-g turn maneuver at constant altitude and velocity, we will attempt perfect tracking of $\boldsymbol{\sigma} = [\phi \ \theta \ \psi]^T, \boldsymbol{\omega} = [p \ q \ r]^T$, or $[\dot{\phi} \ \dot{\theta} \ \dot{\psi}]^T$. Thus, for the definition of $\boldsymbol{\sigma}$ and $\boldsymbol{\omega}$ as just described, we rewrite the equations of motion as follows:

$$\begin{aligned} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} &= \begin{pmatrix} p + q \tan \theta \sin \phi + r \tan \theta \cos \phi \\ q \cos \phi - r \sin \phi \\ q \sin \phi \sec \theta + r \cos \phi \sec \theta \end{pmatrix} \\ \begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} &= \begin{pmatrix} l_\beta \beta + l_q q + l_r r + (l_{\beta\alpha} \beta + l_{r\alpha} r) \Delta \alpha + l_p p - i_1 q r \\ \bar{m}_\alpha \Delta \alpha + \bar{m}_q q + i_2 p r - m_\alpha p \beta + m_\alpha (g_0/V) (\cos \theta \cos \phi - \cos \theta_0) \\ n_\beta \beta + n_r r + n_p p + n_{p\alpha} p \Delta \alpha - i_3 p q + n_q q \end{pmatrix} \\ &+ \begin{pmatrix} l_{\delta a} & l_{\delta r} & 0 \\ 0 & 0 & \bar{m}_{\delta e} \\ n_{\delta a} & n_{\delta r} & 0 \end{pmatrix} \begin{pmatrix} \delta_a \\ \delta_r \\ \delta_e \end{pmatrix} \\ \begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \end{pmatrix} &= \begin{pmatrix} q - p \beta + z_\alpha \Delta \alpha + (g_0/V) (\cos \theta \cos \phi - \cos \theta_0) \\ y_\beta \beta + p (\sin \alpha_0 + \Delta \alpha) - r \cos \alpha_0 + (g_0/V) \cos \theta \sin \phi \end{pmatrix} \end{aligned}$$

The terms corresponding to inertial coupling are further taken out of this system description, and we denote $H(\boldsymbol{\sigma}, \boldsymbol{\omega}) = [-i_1 q r + i_2 p r - i_3 p q]^T$. Thus, the system description without the $\dot{\alpha}$ and $\dot{\beta}$ equations can be cast into the general description in Eqs. (1) and (2). Observe that when $\boldsymbol{\sigma} = \boldsymbol{\sigma}_0 = 0$ and $\boldsymbol{\omega} = \boldsymbol{\omega}_0 = 0$,

$$\dot{\alpha} = z_\alpha \Delta \alpha, \quad \dot{\beta} = y_\beta \beta$$

One can show that the α and β dynamics are locally input-to-state stable, treating p, q , and r as inputs to the respective subsystems. Substituting $p = q = r = 0$, we see that $\Delta \alpha = 0$, and $\beta = 0$ are exponentially stable equilibria as long as z_α and y_β are negative. The

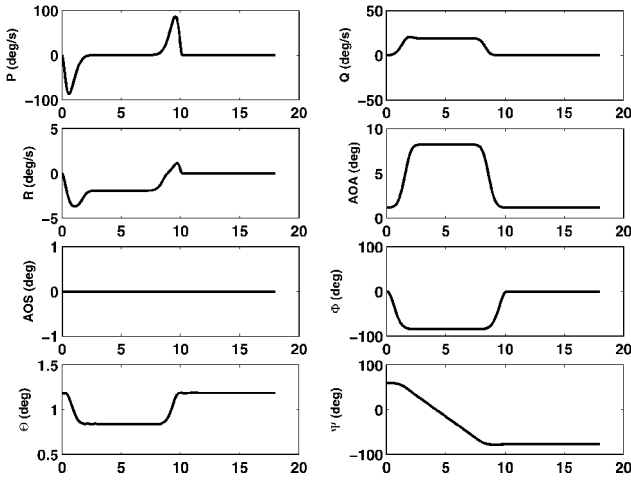
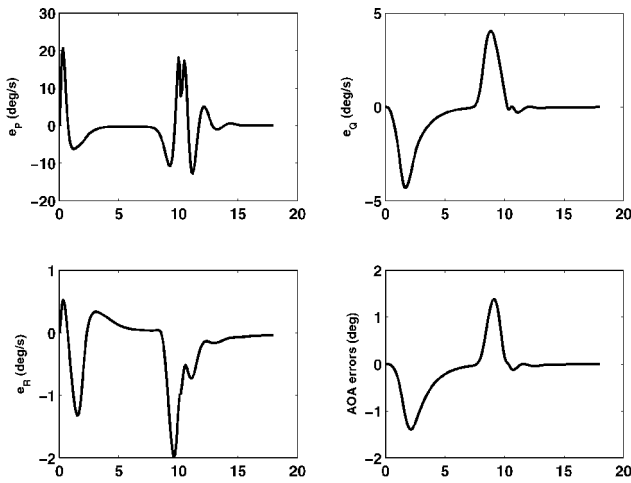
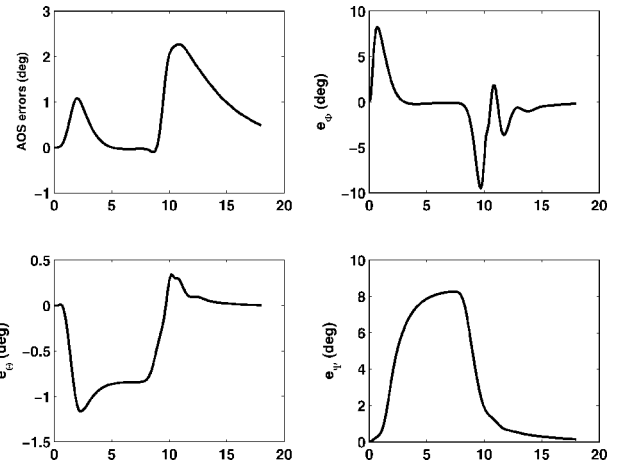
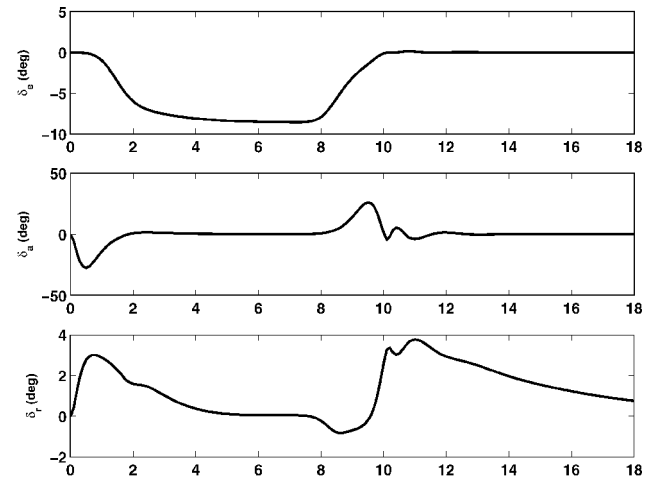
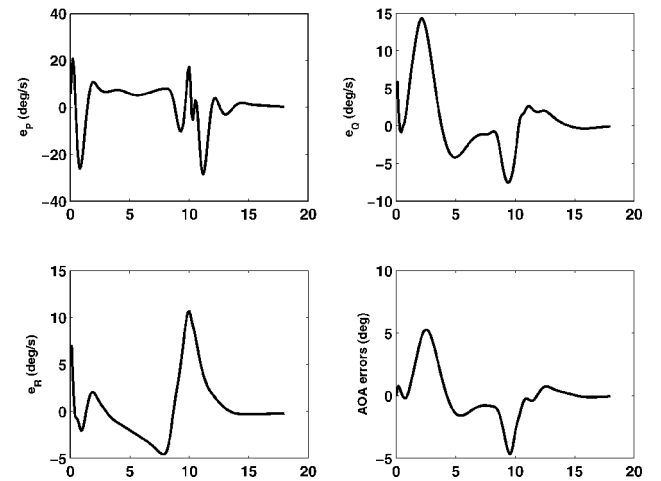


Fig. 1 Open-loop trajectory.

Fig. 2 Errors (P , Q , R , and α) vs time (seconds) [ideal initial conditions (ICs) and no external disturbances].

subsequent analysis to show the local input-to-state stability follows along the same line outlined in Ref. 1. Alternatively, the problem could be posed in the backstepping setting outlined in Refs. 25 and 26 to first stabilize the angle of attack (AOA) and angle of sideslip (AOS) dynamics robustly and then solve the tracking control problem. Note that the maneuver does not involve tracking α and β . We only require that subsystem dynamics be stable. The maneuver is a 9-g, 180-deg heading change at constant altitude and velocity. The reference trajectory is generated with the preceding model. The first phase of the trajectory is a roll-into-the-turn phase, where a desired bank angle is commanded, followed by a steady heading change at constant bank angle. The final phase of the maneuver is the roll-out-of-the-turn, where the wings are brought to straight and level orientation. The open-loop reference trajectory is shown in Fig. 1. The actual system is assumed to be a perturbation of the model used for the trajectory generation. The stability derivatives are all assumed perturbed by as much as 30%, in addition to initial condition errors. A constant external disturbance acceleration of 6 deg/s^2 is added to the roll, pitch, and yaw axes. Figs. 2–4 correspond to ideal initial conditions, that is, the actual system initial conditions are the same as that of the reference trajectory and the external disturbances are absent. We see that, by the end of the maneuver, all of the angular errors and the rate errors converge to zero. There is a very small offset in the angle of sideslip of < 0.5 degrees.

Figures 5–7 correspond to the case where there is a constant external disturbance acceleration in the roll, pitch, and yaw channels, in addition to the initial condition errors. We observe excursions in the α and β responses, as expected. These excursions could be controlled by modifying the reference trajectory values as suggested

Fig. 3 Errors (β , ϕ , θ , and ψ) vs time (seconds) (ideal ICs and no external disturbances).Fig. 4 Controls (δ_e , δ_a , and δ_r) vs time (seconds) (ideal ICs and no external disturbances).Fig. 5 Errors (P , Q , R , and α) vs time (seconds) (IC errors and external disturbances).

in Ref. 24, but we have not done that here. In this case, the body axes rate errors and the AOA and AOS errors converge to zero at the end of the maneuver, but there are nonzero steady-state errors in the roll, pitch, and yaw angles. It is important to arrest the AOA and AOS errors because these could easily couple through the body axes rates and drive the closed-loop system unstable. The maximum angular errors are less than 8 deg. Note that the inclusion of an integral of the angular errors term in the control law could potentially nullify the

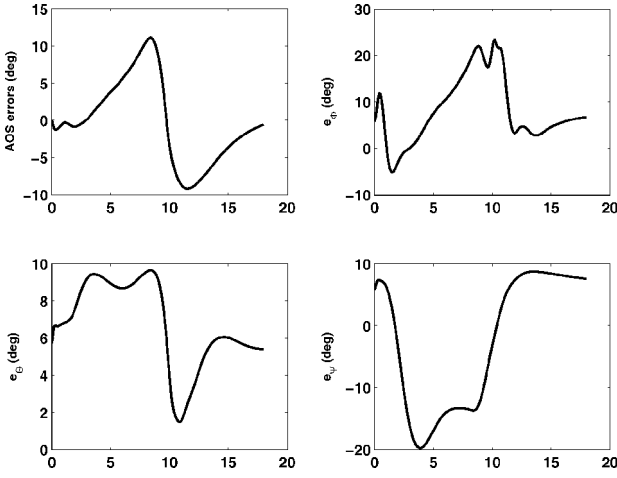


Fig. 6 Errors (β , ϕ , θ , and ψ) vs time (seconds) (IC errors and external disturbances).

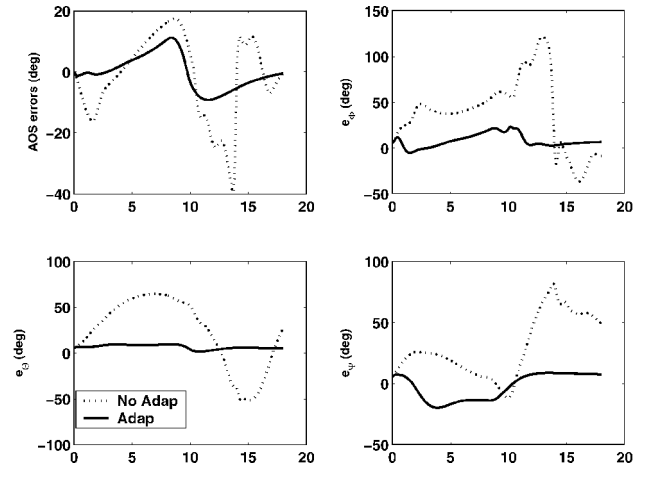


Fig. 9 Errors (β , ϕ , θ , and ψ) vs time (seconds) (IC errors and external disturbances), with/without adaptation.

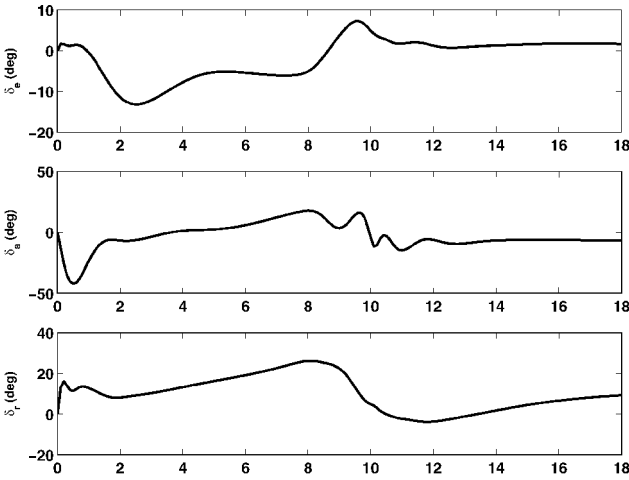


Fig. 7 Controls (δ_e , δ_a , and δ_r) vs time (seconds) (IC errors and external disturbances).

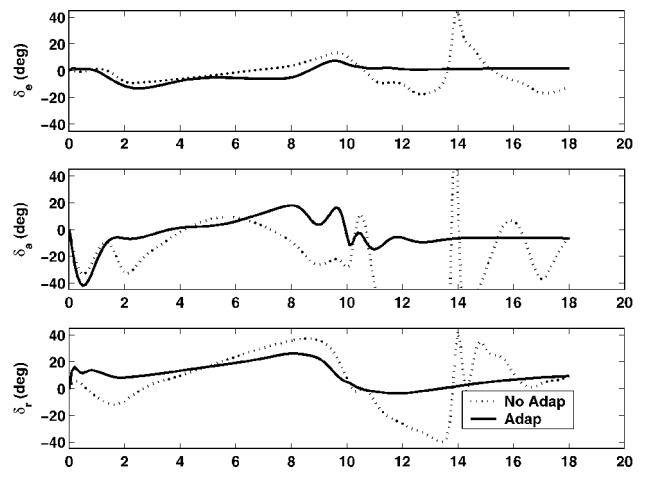


Fig. 10 Controls (δ_e , δ_a , and δ_r) vs time (seconds) (IC errors and external disturbances), with/without adaptation.

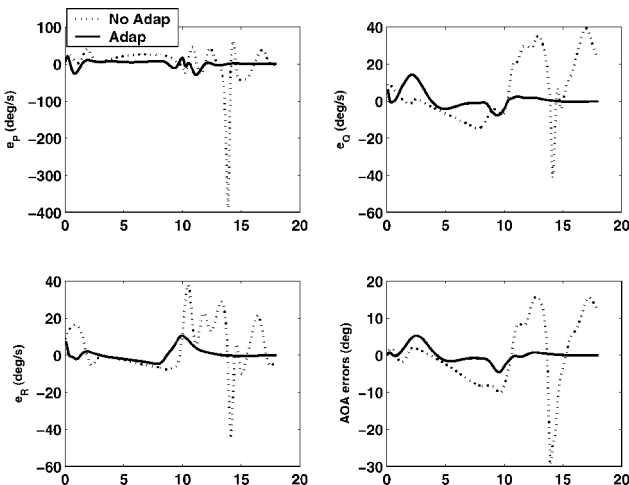


Fig. 8 Errors (P , Q , R , and α) vs time (seconds) (IC errors and external disturbances), with/without adaptation.

steady-state errors. However, the closed-loop stability would have to be reexamined because the adaptation laws would be different. Note that transient performance of the closed-loop system could be improved further by the choice of appropriate adaptation and control law parameters.

Figures 8–10 show the results for the case with and without adaptation. Note that the performance is unacceptable for the case when adaptation is switched off. However, we extract acceptable

performance with adaptation on. This result corresponds to the case where there is 30% uncertainty in the system matrices, initial condition errors, and constant disturbance acceleration in the roll, pitch, and yaw axes. Note that the results without adaptation correspond to a dynamic inversion-type controller. Some amount of stabilization is achieved through the prescription of the Hurwitz matrix A_m . We acknowledge that a more rigorous robust controller could have been designed to extract better tracking performance; however, the objective of this exercise was to show the benefits of adaptation as opposed to a pure dynamic inversion-based controller.

Conclusions

A structured model reference adaptive controller was developed to track aggressive aircraft maneuvers. The control design explicitly incorporates the knowledge of the underlying structure of the differential equations governing the motion of the aircraft. We guarantee globally exponentially stable tracking error dynamics for the known parameter case in the absence of external disturbances. For the case where the parameters are uncertain, the adaptive controller guarantees asymptotic stability of the tracking error dynamics in the absence of external disturbances. In the presence of external disturbances, the tracking errors are shown to be bounded in the mean square small sense. The aircraft model is not exactly in the same form as the formulation requires. However, for the trajectory that is tracked, under suitable assumptions, we extend the theory and show through simulation that, indeed, stable tracking error dynamics are achievable for sufficiently large model errors, initial condition errors, and external disturbances.

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